

Review of Probability and Statistics: Key Terms

Basic Terminology

1. **Random Variables:** $X, Y, Z, \dots$ vs. observations $x, y, z, \dots$
2. **Expectation:** $E(X) = \mu = \sum x f_X(x); E(aX + b) = aE(X) + b;$
3. **Variance:** $Var(X) = \sigma^2 = E(X - \mu)^2 = \sum (x - \mu)^2 f_X(x); Var(aX + b) = a^2 Var(X)$
4. **Standard Deviation:** $\sigma = \sqrt{\sigma^2}, StdDev(aX + b) = \sqrt{a^2 Var(X)} = a StdDev(X)$
5. **Standardize a Random Variable:** $Z = \frac{X - \mu}{\sigma}; E(Z) = 0 \text{ and } Var(Z) = 1$
6. **Joint Density:** $f_{XY}(x, y) = P(X = x \text{ \& } Y = y)$
7. **Independence:** $f_{XY}(x, y) = P(X = x)P(Y = y) = f_X(x)f_Y(y)$ for all x and y
8. **Covariance:** $Cov(X, Y) = \sigma_{XY} = E(X - \mu_X)(Y - \mu_Y) = \sum (x - \mu_X)(y - \mu_Y) f_{XY}(x, y);$
 $Cov(X, X) = \sigma_{XX} = \sigma_X^2 = Var(X), Cov(a + bX, c + dY) = bdCov(X, Y)$
9. If X and Y are independent, then $Cov(X, Y) = \sigma_{XY} = 0$; opposite need not hold
10. **Correlation:** $Corr(X, Y) = \rho_{XY} = \frac{Cov(X, Y)}{StdDev(X) StdDev(Y)} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}, -1 \leq \rho_{XY} \leq 1$
11. **Conditional Density:** $f_{Y|X}(y | x) = P(Y = y | X = x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$
12. If X and Y are independent then $f_{Y|X}(y | x) = f_Y(y)$ and $f_{X|Y}(x | y) = f_X(x)$

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Multiple RV's: X , Y , and $\{X_i\}$

13. **Expectation:** $E(a_0 + \sum a_i X_i) = a_0 + \sum E(a_i X_i) = a_0 + \sum a_i E(X_i)$

14. **Variance:** $Var(X + Y) = \sigma_X^2 + 2Cov(X, Y) + \sigma_Y^2$, $Var\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j Cov(X_i, X_j)$

15. **Independence:** $f_{X_1, \dots, X_n}(x_1, x_2, \dots, x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \dots f_{X_n}(x_n) = \prod_{i=1}^n f_{X_i}(x_i)$

16. If $Cov(X_i, X_j) = 0$ when $i \neq j$, then $Var\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j Cov(X_i, X_j) = \sum_{i=1}^n a_i^2 \sigma_i^2$

17. $Corr(a_1 X_1 + b_1, a_2 X_2 + b_2) = \pm Corr(X_1, X_2)$, $sign = sign(a_1 a_2)$

Normal Distribution

18. **Standard Normal (Gaussian):** $N(\mu, \sigma^2)$ has mean μ and variance σ^2

19. **Independent and Identically Distributed:** *iid*

20. If $\{X_i\}$ *iid* $N(\mu, \sigma^2)$ and $Y = \sum \alpha_i X_i$, then Y will have a Normal Distribution, with

$$E(Y) = \sum \alpha_i \mu = \mu \sum \alpha_i \text{ and } Var(Y) = \sum \alpha_i^2 \sigma^2 = \sigma^2 \sum \alpha_i^2.$$

21. If $\{X_i\}$ *iid* $N(\mu, \sigma^2)$ and $\bar{X} = \frac{1}{n} \sum X_i$, then $\bar{X} \sim N(\mu, \frac{1}{n} \sigma^2)$ and $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$